On Technical Analysis

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Technical analysis, or the use of past prices to infer private information, has value in a model in which prices are not fully revealing and traders have rational conjectures about the relation between prices and signals. A two-period dynamic model of equilibrium is used to demonstrate that rational investors use historical prices in forming their demands and to illustrate the sensitivity of the value of technical analysis to changes in the values of the exogenous parameters.

It is well known that current spot prices of traded assets provide information about future spot prices when market participants are heterogeneously informed. However, spot prices generally are imperfect aggregators of private information. For example, if the current spot price depends on the unobserved current supply of the good as well as on the private information of market participants, then it is not a sufficient statistic for the private information. Consequently, market participants use their private signals in addition to the observed price in forming their demands.

Noise in the current equilibrium spot price also makes it impossible for that price to reveal perfectly the private information from earlier periods. As a result, historical prices together with current prices allow more accurate inferences about past and present signals than do cur-

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rent prices alone. Because current spot prices are not fully revealing, past prices, that is, technical analysis, provide information to agents forming their demands.

Consider a three-date model in which the time 1 aggregate supply of the risky asset is uncertain, and investors receive private signals about the time 3 payoff of the asset.[1] In this setting, an individual is unable to infer the average value of all investors' signals (a sufficient statistic for the aggregate information set) by observing the time 1 price and his own private signal. Suppose that investors receive additional information at time 2. If the investors' time 1 signals remain private, and the time 2 aggregate supply also is uncertain, then each investor will find it impossible to infer the average time 1 or time 2 signal from the time 2 price. The time 1 price is useful in learning about the aggregate information set because it is not perturbed by the noisy variation in the time 2 supply; however it also is not influenced by time 2 signals. Hence, inferences about the signals from either the time 1 price or the time 2 price alone are strictly dominated by inferences considering both prices.

Individuals employ technical analysis (TA) even though the time 2 price is set competitively by rational investors using all public information, including the time 1 price. One implication is that financial markets are not weak-form efficient in the sense that the current price reflects all information contained in past prices. However, the degree to which forecasts of future spot prices are improved by the use of TA remains an open question.

Hellwig (1982), Singleton (1985), and Grundy and McNichols (1989) also develop models in which historical prices are useful to investors in equilibrium. In Hellwig's model, investors are constrained from using current price in forming their demands and must use the most recent past price as a substitute. Additional historical prices are not useful. Singleton studies the stationary, temporal behavior of asset prices in an economy with an infinity of trading dates and myopic investors. He finds that the time-series properties of asset prices are similar across two alternative economies: one with heterogeneously informed investors and the other with homogeneously, partially informed investors. Because TA has no value when investors are homogeneously informed, our analysis demonstrates that investment demand may be sensitive to the distinction between homogeneous and heterogeneous information even if price behavior is not. Grundy and McNichols examine a two-period economy in which investors receive correlated, private signals at time 1 and a time 2 public signal. They demonstrate existence and examine properties of linear, rational price functions when there is no time 2 variation in supply, and they discuss the effects of the addition of a second supply variation. In our model, investors' time 1 and 2 signals are private and uncorrelated. Furthermore,

[1] This is analogous to the single-period setting of Diamond and Verrecchia (1981) and Hellwig (1980), except that there also is trading at date 2.

we focus exclusively on the case in which supply variations occur in each period.

In the following section, a dynamic, two-period model of asset prices is introduced. Alternative economies of rational and myopic investors are presented. Equilibria are shown to exist generally for the myopic-investor economy and for special cases of the rational-investor economy. Equilibria of the rational-investor economy are analyzed numerically in Section 2. The form of the resulting prices as functions of exogenous information allows a discussion of the rationale for TA. Implications of the analysis are compared to extant definitions of weak-form efficiency in Section 3. Section 4 summarizes and concludes the article.

1. Equilibrium in a Dynamic Economy with Endogenous Beliefs

In this section, the Hellwig (1980) and Diamond and Verrecchia (1981) noisy rational expectations models are extended to two periods. A riskless asset and one risky asset are exchanged in markets opening at times $t = 1$ and $t = 2$. Consumption occurs only at $t = 3$ when each share of the riskless asset pays 1 unit and the risky asset provides a random payoff of u. The riskless rate is assumed to be 0.[2] Investors i ($i = 1, 2, 3, \ldots$) are a priori identical and countably infinite in number. Each enters the first period with n_0 units of the riskless asset,[3] and chooses a feasible trading strategy to maximize the expected utility of consumption at time 3:

$$E[-\exp(-Rw_{i3}) \mid \Xi_0] \tag{1}$$

where R is the common absolute risk aversion parameter and Ξ_0 is the common prior information available to traders. Just prior to the opening of the market at time t, each individual receives a private signal, y_{it}, of the time 3 payoff of the risky asset. Competitive trading establishes the risky asset price P_t at each date. The information available to investor i at time 1 and 2 is $\Xi_{i1} \equiv \{\Xi_0, y_{i1}, P_1\}$ and $\Xi_{i2} \equiv \{\Xi_{i1}, y_{i2}, P_2\}$, respectively.

A feasible trading strategy requires that planned asset holdings be measurable with respect to the trader's available information and satisfy the individual's budget at each trading date. Let d_{it} denote individual i's time t holding of the risky asset. Then the payoff at time 3 is $n_0 + d_{i1}(P_2 - P_1) + d_{i2}(u - P_2)$ and the optimal trading strategy is determined by sequentially solving

$$J_{i2}(d_{i1}) \equiv \max_{d_{i2}} E\{-\exp[-R(n_0 + d_{i1}(P_2 - P_1) + d_{i2}(u - P_2))] \mid \Xi_{i2}\} \tag{2a}$$

[2] Because consumption occurs only at the final date, investors' marginal utilities at $t = 1$ and $t = 2$ are indeterminate without an exogenous specification of the riskless rate of interest.

[3] As in Hellwig (1980), the (possibly random) endowments of shares of the risky asset are left unspecified. This assumption implies that individuals ignore information the random endowments might provide. In the multiperiod model developed here, the assumption also implies that individuals do not use the risky asset to hedge against future variations in wealth resulting from the random endowment. Brown and Jennings (1988) allow random individual endowments of the risky asset.

and

$$J_{i1} \equiv \max_{d_{i1}} E\{J_{i2}(d_{i1}) \mid \Xi_{i1}\} \tag{2b}$$

Define $\Xi_t \equiv (\Xi_{1t}, \Xi_{2t}, \ldots)$ as the information set available to the market at time t. A rational expectations equilibrium is a pair of demand functions (d_{i1}, d_{i2}) for each investor and a pair of equilibrium price functions (P_1, P_2) that together satisfy the following conditions. First, the P_t are functions of Ξ_t through their dependence on investors' demands and per capita supplies,[4] and, for each realization of Ξ_t, traders' price conjectures are identical to $P_t(\Xi_t)$. Second, each trader's strategy is feasible and solves Equations (2), when the conjectured price functions are used in Equation (2a). Finally, traders' strategies and the equilibrium prices are such that markets clear. Define $d_t \equiv \lim_{I\to\infty} \Sigma_{i=1}^{I} d_{it}/I$, and let x_1, and $x_1 + x_2$ denote the random per capita supplies of the risky asset at times 1 and 2, respectively.[5] The market-clearing condition is written

$$x_1 + x_2 = d_2 \tag{3a}$$

and

$$x_1 = d_1 \tag{3b}$$

where these equalities hold with probability 1.

The exogenous random variables in the economy (the asset supplies and payoffs and the private signals received by investors) are assumed to follow a multivariate normal distribution. Normality of the distribution of the conjectured prices follows from their linear dependence on the exogenous variables in a rational expectations equilibrium.

Individuals' homogeneous prior beliefs about u are represented by a normal distribution with mean y_0 and variance h_0. The private signal observed by individual i at time t is

$$y_{it} = u + \epsilon_{it} \tag{4}$$

where $\epsilon_{it} \sim N(0, s_t)$ and $E(u\epsilon_{it}|\Xi_0) = 0$. The signals' errors are independent across investors and time periods. Because the ϵ_{it} are distributed normally with finite variances homogeneous across investors, the law of large numbers implies that average signal, $y_t \equiv \lim_{I\to\infty} \Sigma_{i=1}^{I} y_{it}/I$, equals u with probability 1 for each t. The per capita supply increment x_t is distributed $N(0, V_t^2)$ conditional on Ξ_0, with x_t independent of u and the private signals. The correlation between the supply increments x_1 and x_2 is denoted ρ.

The stochastic behavior of P_t depends on its functional relationship to the exogenous variables x_t and u. Individuals conjecture that prices are

[4] See Diamond and Verrecchia (1981) for a discussion of this issue.

[5] Noise is necessary for prices to be less than fully revealing of private information. Following Grossman and Stiglitz (1980), Diamond and Verrecchia (1981), and Hellwig (1980), we introduce noise via systematic variation in supply.

linear functions of the supply increments and aggregate information:

$$P_2 = \alpha_2 y_0 + \beta_2 u - \gamma_2 x_1 - \delta_2 x_2 \tag{5a}$$

$$P_1 = \alpha_1 y_0 + \beta_1 u - \gamma_1 x_1 \tag{5b}$$

Conjectures are identical across individuals and the coefficients are determined in an equilibrium in which the conjectures are rational. Because the price conjectures (5) are linear functions of normal variates, they are normally distributed.

1.1 A linear noisy rational expectations equilibrium

Appendix A derives the system of equations that equates the coefficients of Equations (5) with the corresponding coefficients in the price functionals satisfying Equations (3), assuming a noisy rational expectations equilibrium exists. This derivation is sketched below.

Given the price conjectures (5), the risky asset demands planned by trader i to solve Equations (2) can be written

$$d_{i2} = \frac{E(u \mid \Xi_{i2}) - P_2}{R\mathrm{var}(u \mid \Xi_{i2})} \tag{6a}$$

$$d_{i1} = \frac{E(P_2 \mid \Xi_{i1}) - P_1}{RG_{11}} + \frac{E(d_{i2} \mid \Xi_{i1})(G_{12} - G_{11})}{G_{11}} \tag{6b}$$

$\mathrm{Var}(u|\Xi_{i2})$, G_{11}, and G_{12} are computed in Appendix A using the prescribed covariances of the exogenous variables and the coefficients of the price functions (5). They are constants, identical across investors. Averaging equations (6) over i and imposing the market-clearing conditions (3) provides the potential equilibrium price functions:

$$P_2 = \mu_2 - R\sigma_2^2(x_1 + x_2) \tag{7a}$$

and

$$P_1 = \eta + \frac{(G_{12} - G_{11})(\mu_1 - \eta)}{\sigma_1^2} - RG_{11}x_1 \tag{7b}$$

where $\mu_t \equiv \lim_{I\to\infty} \Sigma_{i=1}^{I} E(u|\Xi_{it})/I$, for $t = 1, 2$

$\eta \equiv \lim_{I\to\infty} \Sigma_{i=1}^{I} E(P_2|\Xi_{i1})/I$

$\sigma_t^2 \equiv \mathrm{var}(u|\Xi_{it})$, for $t = 1, 2$

Because μ_t and η are linear functions of y_0, u, and x_t, Equations (7) may be written as

$$P_2 = \hat{\alpha}_2 y_0 + \hat{\beta}_2 u - \hat{\gamma}_2 x_1 - \hat{\delta}_2 x_2 \tag{8a}$$

and

$$P_1 = \hat{\alpha}_1 y_0 + \hat{\beta}_1 u - \hat{\gamma}_1 x_1 \tag{8b}$$

where the coefficients (with carats) can be expressed as functions of the coefficients in Equations (5) (without carats). A rational expectations equilibrium with linear price functions exists for the rational-investor economy if there exists a solution to the set of simultaneous equations relating the coefficients of Equations (8) to their counterparts in Equations (5).

Existence can be demonstrated for limiting cases of the rational-investor economy in which a fully revealing equilibrium price exists at time 2. Assuming such a price, one finds that $\beta_2 = 1$, $\alpha_2 = \gamma_2 = \delta_2 = 0$, $\beta_1 = (s_1 R^2 V_1^2 h_0 + h_0)/[s_1 R^2 V_1^2(h_0 + s_1) + h_0]$, $\alpha_1 = 1 - \beta_1$, and $\gamma_1 = s_1 R \beta_1$ represent an equilibrium. When the time 2 price is fully revealing, the time 1 price coefficients are identical to those in Hellwig's (1980) single-period model.

Sufficient conditions for the time 2 price to be fully revealing are that the private signals at time 1 or 2 are noiseless ($s_1 = 0$ or $s_2 = 0$) or that $\rho = -1$ and $V_1^2 = V_2^2$. In the first case, u is fully revealed by each investor's time 2 private information set. In the second, the aggregate supply disturbance is identically zero so P_2 is not perturbed by noise. In either case, the time 2 price is fully revealing regardless of whether or not investors recall the historical price P_1.

Under the following alternative conditions P_2 is not fully revealing, but P_1 and P_2 are jointly fully revealing. First, suppose that $V_2^2 = 0$ and the other variances are finite and nonzero. This implies that $x_2 \equiv 0$ and, as may be seen from the correctly conjectured equilibrium price functionals (5), P_1 and P_2 jointly reveal u and x_1. Alternatively, suppose that the second-period supply increment is proportional to the first period's supply increment: $x_2 = qx_1$. Then, as before, P_1 and P_2 jointly reveal u and x_1. In either of these cases, $P_2 = u$ to avoid arbitrage.[6]

In the one-period models of Grossman (1976) and Grossman and Stiglitz (1980) without a random per capita supply, equilibrium price reveals a sufficient statistic for the asset payoff. As is well known, this implies that each investor's signal is redundant so that there is no incentive for investors to condition their beliefs and demands on their private information. In our model, if P_1 and P_2 jointly reveal u, then the absence of arbitrage implies that $P_2 \equiv u$, that is, current price fully reveals the payoff u. Yet current price P_2 is fully revealing only because the two prices jointly reveal u and x_1; when historical price is forgotten by every investor, current price is not fully revealing. Thus, there is no incentive for any individual to expend resources to observe past price (which is public information) because each investor is conditioning on past price.

Beyond the special cases discussed above, there are no results establishing existence of an equilibrium in the rational-investor economy. For this reason, we turn to a more simple economy in which investors behave myopically. General existence conditions are established for this economy.

[6] Equilibrium price coefficients also can be derived when there is no time 2 signal ($s_2^{-1} = 0$) and there is no time 2 supply shock ($V_2^2 = 0$); see Grundy and McNichols (1989) for this result.

1.2 Equilibrium in a myopic-investor economy

The myopic-investor economy is identical to that presented in the previous section except that the first-period demands

$$d_{i1}^{m} \equiv \frac{E(P_2 \mid \Xi_{i1}) - P_1}{R\mathrm{var}(P_2 \mid \Xi_{i1})} \tag{6c}$$

are substituted for the demands (6b) in the determination of the market-clearing price (8b).[7] Note that the primary difference in demands (6c) and (6b) is the elimination of a hedging demand that is proportional to the time 1 expectation of time 2 demand. Subject to this difference, the equilibrium price coefficients are determined in the myopic-investor economy as they are in the rational-investor economy. As others, for example, Singleton (1985), have found, the elimination of the investors' hedging demands simplifies the analysis. Indeed, it is possible to prove

Theorem 1. *Given that V_1^2, V_2^2, h_0, s_1, s_2, and R are each greater than zero and that $|\rho| < 1$, a linear, noisy rational expectations equilibrium exists in the myopic-investor economy, and each of the price coefficients other than (possibly) α_1 and γ_2 is nonzero.*

Proof. The proof proceeds by deriving expressions for the coefficients α_2, β_2, γ_2, and δ_2 of Equation (5a) as functions of the ratio $Z \equiv \beta_1/\gamma_1$. Substituting these expressions into $\hat{\beta}_1$ and $\hat{\gamma}_1$ of Equation (8b) provides a fifth-degree polynomial in Z [defined by Equation (A24) of Appendix B]. An equilibrium exists if and only if there exists a finite zero to this polynomial satisfying $\delta_2 \neq 0$. A solution, not necessarily unique, is shown to exist such that α_2, β_2, and δ_2 are nonzero. The details are provided in Appendix B. ■

1.3 The conditions for technical analysis to have value

Risky asset demands in Equations (6) depend on the contemporaneous price and conditional moments of u. The linearity of $\mu_{i2} \equiv E(u|\Xi_{i2})$, $\mu_{i1} \equiv E(u|\Xi_{i1})$, and $E(P_2|\Xi_{i1})$ as functions of the elements of the information sets Ξ_{i2} and Ξ_{i1} allows demands to be written as linear functions of those elements:

$$d_{i2} = \Psi_{21}y_0 + \Psi_{22}y_{i1} + \Psi_{23}y_{i2} + \Psi_{24}P_1 + \Psi_{25}P_2 \tag{9a}$$

and

$$d_{i1} = \Psi_{11}y_0 + \Psi_{12}y_{i1} + \Psi_{13}P_1 \tag{9b}$$

These linear forms obtain whether investors are myopic or rational, although the values of the coefficients differ in the two economies.

TA has no value in equilibrium if and only if the investor's time 2 beliefs

[7] The formal results in Appendix A are applicable to the myopic-investor economy except for the derivation of the time 1 demands in Proposition A4 and the derivation of the time 1 price coefficients in Proposition A6.

and demands do not vary with P_1 after conditioning on the other variables in Ξ_{i2}; that is, if and only if $\Psi_{24} = 0$. Equation (6a) implies that $\Psi_{24} = 0$ if and only if the investor's time 2 expectation of the risky asset payoff μ_{i2} does not vary with P_1 and, hence, with x_1. In other words, TA has no value if and only if $\text{cov}(u, P_1 | P_2, y_{i2}, y_{i1}, \Xi_0) = 0$. Equations (7a) and (8a) imply that μ_{i2} does not vary with x_1 if and only if the time 2 price coefficient on x_1, γ_2, is equal in value to the coefficient on x_2, δ_2. Thus, the conditions, (1) $\Psi_{24} = 0$, (2) $\text{cov}(u, P_1 | P_2, y_{i2}, y_{i1}, \Xi_0) = 0$, (3) $\gamma_2 = \delta_2$, and (4) TA has no value, are equivalent.

Using expressions in Appendix A for β_2, γ_2, and δ_2 as functions of $Z \equiv \beta_1/\gamma_1$, it can be shown that $\gamma_2 = \delta_2$ and that TA has no value if and only if

$$\frac{\beta_2}{\gamma_2} = \frac{s_2 + s_1}{R s_1 s_2} \tag{10a}$$

and

$$\frac{\beta_1}{\gamma_1} = \frac{(s_1 + s_2) V_1 (V_1 + \rho V_2)}{R s_1 s_2 (V_1^2 + \rho V_1 V_2 + V_2^2)} \tag{10b}$$

Condition (10b) defines a subset of the parameter space in which TA has no value, while (10a) follows from (10b) and the necessary conditions for an equilibrium. Conditions (10) permit us to make a strong statement regarding the value of TA in the myopic-investor economy.

Theorem 2. *Under the parametric restrictions of Theorem 1, TA has value in every linear, two-period rational expectations equilibrium of the myopic-investor economy.*

Proof. See Appendix B. ■

This result follows from a demonstration that the ratio (10b) defining Z is not a zero of the fifth-degree polynomial equilibrium condition.

In the following section is a numerical analysis of the rational-investor economy equating the coefficients of Equations (8) with those in Equations (5). The results demonstrate the existence of equilibria in which Equations (10) do not obtain and TA is of value. The results also quantify this value for a range of parameter values.

2. The Value of Technical Analysis in the Rational-Investor Economy

The value of TA is assessed in two ways. First, the relation between the optimal time 2 demand for the risky asset by an individual and the time 1 price is examined. Second, the value to an otherwise uninformed investor of observing the time 1 price is computed. The equilibrium price distributions are held constant as this comparison is made in order to provide a measure of the private value of technical analysis.

The base values of the parameters for the numerical analysis are listed in Appendix C, Section A. The values for the moments are consistent with a sample distribution of monthly spot wheat prices taken from the *Statistical Annual 1978 Chicago Board of Trade.* Appendix C also shows the coefficients of the equilibrium price functions in Section B, and the coefficients of the demand functions (9) in Section C, implied by the base parameter values. Predictably, time 2 demand is a decreasing function of the contemporaneous price P_2. The coefficient of P_1 in the time 2 demand function is -0.06. Thus, a decrease in P_1, like an increase in P_2, is "good news" in the sense of Milgrom (1981); this is confirmed by the negative dependence of μ_{i2} on P_1 shown in Section D.[8]

To better understand this result, note that[9]

$$\begin{aligned}\operatorname{cov}(u, P_1 \mid y_{i1}, y_{i2}, P_2, \Xi_0) &= \beta_1 \operatorname{var}(u \mid y_{i1}, y_{i2}, P_2, \Xi_0)\\ &\quad - \gamma_1 \operatorname{cov}(u, x_1 \mid y_{i1}, y_{i2}, P_2, \Xi_0)\\ &\equiv \beta_1 V_u - \gamma_1 C_{ux} \qquad (11)\end{aligned}$$

In our example, the values of β_1, γ_1, β_2, γ_2, and C_{ux} are positive. If the conditional covariance of u and P_1 were also positive, then observing a large value of P_1 would be good news and the investor's time 2 risky asset demand would increase. In our example, however, the conditional covariance is negative given the base parameter values, so that the investor demands less of the risky asset at time 2 conditional on a high time 1 price. A high P_1 is viewed as bad news conditional on P_2 because $\gamma_1 C_{ux} > \beta_1 V_u$, which implies that the investor believes a large P_1 is more likely due to small supply than to favorable information about u. Hence, because γ_2, $\beta_2 > 0$, he revises downward the inferences about u initially made conditional on P_2.

Now consider an uninformed, price-taking investor arriving in the market at time 2 and observing only the current price P_2, the prior information Ξ_0, and, possibly, the historical price, P_1. This investor receives no private signal and can be considered a "technician," as opposed to a "fundamental investor."[10] Prices are determined by the infinite number of rational fundamental investors described in Section 1.1.

Given constant absolute risk aversion R, the maximum number of units of the riskless asset the technician will pay to observe the historical price

[8] This result is similar to one of Admati (1985). She demonstrates in a single-period framework with multiple assets that a price increase may be associated with "bad news."

[9] Although u and x_1 are unconditionally uncorrelated, they are correlated conditional on P_2. Provided that β_2 and γ_2 are positive, the equilibrium price function (5a) implies that a high (low) u must be accompanied by a high (low) x_1, holding P_2 and x_2 constant. The numerical results demonstrate a positive $\operatorname{cov}(u, x_1 \mid y_{i1}, y_{i2}, P_2, \Xi_1)$.

[10] The informative value of P_1 is greater than it would be if the technician received private information. Nonetheless, the previous results of this section demonstrate that a rational investor who observes y_{i1} and y_{i2} is willing to pay to observe P_1 at time 2.

in addition to Ξ_0 and P_2 (observed costlessly), is[11]

$$\pi = \frac{1}{R}\log\left[\frac{\operatorname{var}(u \mid P_2, \Xi_0)}{\operatorname{var}(u \mid P_1, P_2, \Xi_0)}\right]^{1/2} \tag{12}$$

An interpretation of the size of π is provided by examination of the correlation of u and P_1 conditional on P_2 and Ξ_0. The difference between the variance of u for the technician conditional on P_1 and that conditional on both P_1 and P_2 can be written

$$\operatorname{var}(u \mid P_2, \Xi_0) - \operatorname{var}(u \mid P_1, P_2, \Xi_0) = \frac{\operatorname{cov}(u, P_1 \mid P_2, \Xi_0)^2}{\operatorname{var}(P_1 \mid P_2, \Xi_0)}$$

Equality (12) and the definition of the correlation coefficient provide

$$\operatorname{Corr}(u, P_1 \mid P_2, \Xi_0)^2 = 1 - \exp(-2R\pi)$$

Thus, π increases in the absolute level of the conditional correlation between u and P_1.

Figures 1, 2, and 4 plot π as a proportion of $E(d_{\tau 2}P_2 \mid \Xi_0)$, the unconditional expected value of the technician's time 2 risky asset position. Because π and $E(d_{\tau 2}P_2 \mid \Xi_0)$ are proportional to risk aversion and are units of the risk-free asset, their ratio is independent of R and is unitless. For our base case, a technician would be willing to spend up to 0.127% of the value of his expected risky asset investment on TA. If the expected risky asset investment were \$100,000, for example, then the technician would be willing to spend \$127 to observe historical price.

Figure 1 shows that a decrease in the variance of the historical fundamental information s_1 and/or the variance of the current fundamental information s_2 leads to an increase in the value of TA over most of the range of the variances examined. This relationship is reversed, however, for relatively small values of either variance.

When s_1 (s_2) is large relative to its base value, the fundamental investors' demands place little weight on y_{i1} (y_{i2}) because the signal is relatively uninformative. Indeed, prices become jointly nearly uninformative of u as s_1 and s_2 approach infinity. Accordingly, the technician is unwilling to pay much for past price in these circumstances. As either s_1 or s_2 declines, fundamental investors' demands place increasing weight on private information, and β_2 increases while γ_2 and δ_2 decrease. An implication of the decline in δ_2 is that the current and the historical prices become *jointly* more revealing of the asset payoff u and the historical supply increment x_1. A second, opposing effect is that the current price *alone* becomes more revealing of the asset payoff as β_2 becomes large. This implies that the observance of P_1 is not very useful when s_1 or s_2 approaches zero; in the

[11] See equation (3.1) of Admati and Pfleiderer (1986) and their associated discussion.

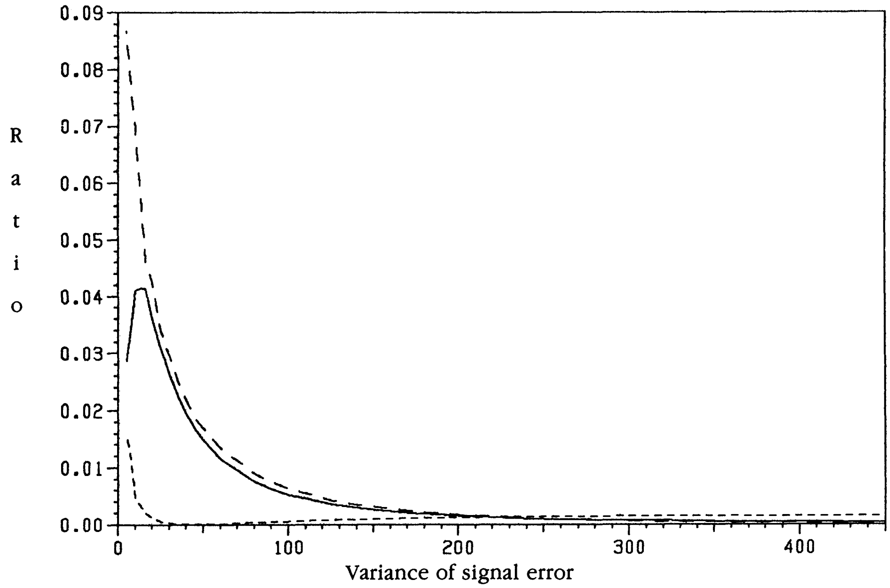

Figure 1
The ratio of the value of technical analysis and expected time 2 investment of the technician, $\pi/E\{d_{T2}\cdot P_2|\mathbb{Z}_0\}$, shown as a function of the variance of the supply increment, V_t^2. The solid line represents the case in which both V_1^2 and V_2^2 are varied, the short-dashed line the case in which only V_1^2 is varied, and the long-dashed line the case in which only V_2^2 is varied.

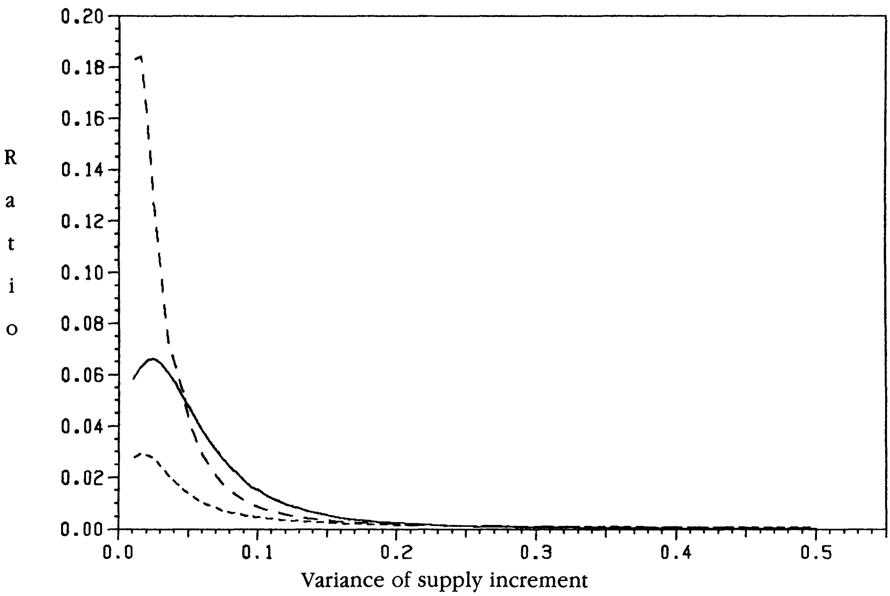

Figure 2
The ratio of the value of technical analysis and expected time 2 investment of the technician, $\pi/E\{d_{T2}\cdot P_2|\mathbb{Z}_0\}$, shown as a function of the variance of the noise, s_t, in private signals, y_{it}. The solid line represents the case in which both s_1 and s_2 are varied, the short-dashed line the case in which only s_1 is varied, and the long-dashed line the case in which only s_2 is varied.

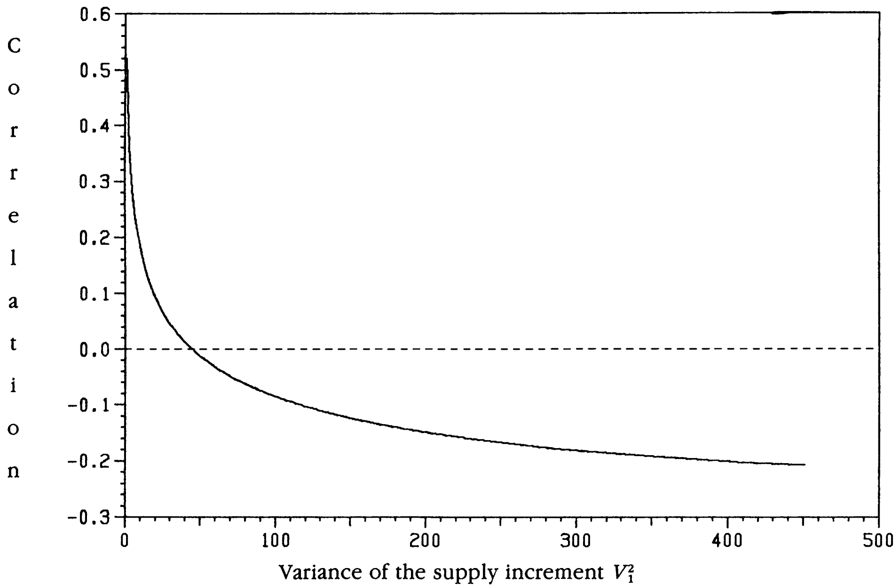

Figure 3
The conditional correlation of u and P_1, corr(u, $P_1 | P_2, \Xi_0$) as a function of V_1^2, the variance of the time 1 per capita supply increment x_1. The values of the other parameters are set equal to the base values shown in Section A of Appendix C.

limit, $P_1 = P_2 = u$ ($P_2 = u$) when $s_1 = 0$ ($s_2 = 0$). It appears from Figure 1 that at moderate to high initial levels of signal variance, the first of these two effects dominates and the value of TA increases as s_1 or s_2 falls. The second effect dominates for low levels of signal variance so that the value of TA declines as the precision of the fundamental information increases.

Figure 2 shows changes in the value of TA due to changes in the variances of the supply increments, that is, changes in noise. It is evident that changes in the value of TA are directly related to changes in historical noise, V_1^2, for large V_1^2 and inversely related for very small V_1^2. The relation for the technician analogous to Equation (11), that is,

$$\text{cov}(u, P_1 | P_2, \Xi_0) = \beta_1 \text{var}(u | P_2, \Xi_0) - \gamma_1 \text{cov}(u, x_1 | P_2, \Xi_0) \quad (11a)$$

helps in understanding the relationship between V_1^2 and the value of TA.

For relatively large values of V_1^2, variations in P_1 are primarily due to variations in x_1. Hence cov(u, $x_1 | P_2, \Xi_0$) is relatively large and, as shown in Figure 3, corr(u, $P_1 | P_2, \Xi_0$) < 0. The technician finds high (low) values of P_1 to be bad (good) news. When V_1^2 is small the conditional correlation of u and P_1 is positive because x_1 has little influence; high values of P_1 are good news. With intermediate values of V_1^2, that is, those near $V_1^2 = 45$, the value of TA is small. For values in this range, it is difficult to discriminate between variations in P_1 to u and those due to x_1 conditional on observing P_2.

Unlike variations in V_1^2, variations in V_2^2 and $V_1^2 = V_2^2$ from the base values

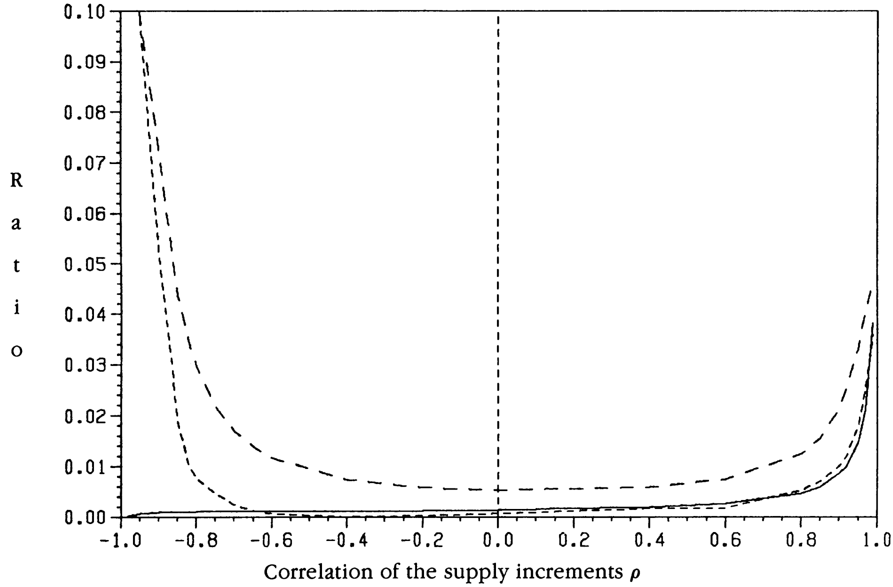

Figure 4
The ratio of the value of technical analysis and expected time 2 investment of the technician, $\pi/E(d_{T2}\cdot P_2\,|\,\Xi_0)$, shown as a function of ρ, the correlation of the supply increments x_1 and x_2. The solid line represents the case in which $V_1^2 = V_2^2 = 226$, the short-dashed line the case in which $V_1^2 = 113$ and $V_2^2 = 226$, and the long-dashed line the case in which $V_1^2 = 226$ and $V_2^2 = 113$. The values of the remaining parameters are set equal to the base values shown in Section A of Appendix C.

provide no cases in which $\text{corr}(u, P_1\,|\,P_2, \Xi_0) = 0$. As documented by Figure 2, a rise in the current noise, V_2^2, decreases the value of TA, independently of the initial value of V_2^2. When V_2^2 is small, neither the current nor the historical price is perturbed significantly by x_2 so the prices jointly reveal the asset payoff and the time 1 supply increment x_1 with great accuracy. TA has considerable value. This is to be contrasted with the limiting case $V_2^2 = 0$ in which the current price alone reveals the payoff and TA has no value. When the noisy variation in P_2 due to x_2 is large, there is little to gain from observing P_1 because the technical analyst still must infer three unknowns from only two prices. Furthermore, because the risk-averse investors' time 1 demands, (6b), are inversely related to the variance of P_2, the sensitivity of P_1 to changes in private information is reduced as V_2^2 rises. In the limit, as V_2^2 goes to infinity, it is impossible to learn anything about u from P_1 because the investors refuse to act on their private information at time 1.

Note that as $V_1^2 = V_2^2$ declines from relatively low levels, the value of TA declines. This is to be contrasted with the increase in the value of TA as either V_1^2 or V_2^2 declines individually. One finds that the current price is highly correlated with both the asset payoff and the historical price when both the current and historical noises are small. Hence there is little left to learn from the historical price once the current price is observed. In the limit, as V_1^2 and V_2^2 jointly approach zero, P_2 alone reveals u, and P_1 is worthless to the technician.

Figure 4 illustrates variations in the value of TA as ρ is altered. For extreme values of the correlation between the supply shocks, $|\rho| = 1$, per capita supplies are identified by a single variable, that is, $x_2 = qx_1$ for some constant q, and current price is perfectly revealing; TA has no value. Nonetheless, provided that $x_1 + x_2$ does not approach zero, the value of TA increases as $|\rho|$ *approaches* 1. This follows from the fact that the three unknowns u, x_1, and x_2 are revealed with great accuracy by the joint use of current and historical price. One sees from Figure 4 that the value of TA does increase as $|\rho|$ approaches 1 when $V_1^2 \neq V_2^2$ and as ρ approaches 1 when $V_1^2 = V_2^2$. In the case $V_1^2 = V_2^2$, the time 2 per capita supply goes to zero (almost surely) as ρ approaches -1 and P_2 becomes nearly perfectly revealing of u leaving little to be learned from the historical price.

Historical attempts to judge the value of TA generally were based on examinations of the unconditional moments of asset returns or price changes. We have shown that, in this model, the value of TA is zero if and only if the *conditional* covariance $\text{cov}(u, P_1 \mid P_2, \Xi_0)$ is zero. It is of interest, therefore, to examine the relation between this covariance and the unconditional moments of the price changes. The definition of covariance and the law of iterated expectations allows one to write the unconditional covariance of price changes as

$$\begin{aligned}\text{cov}(u - P_2, P_2 - P_1 \mid \Xi_0) &= E[\text{cov}(u - P_2, P_2 - P_1 \mid P_2, \Xi_0) \mid \Xi_0] \\ &\quad + \text{cov}[E(u - P_2 \mid P_2, \Xi_0), \\ &\qquad E(P_2 - P_1 \mid P_2, \Xi_0) \mid \Xi_0] \\ &\equiv A + B\end{aligned}$$

The normality of the random variables implies $A = -\text{cov}(u, P_1 | P_2, \Xi_0)$. Hence, one may examine the unconditional covariance of price changes to determine whether TA has value only if the covariance of conditional expected price changes, B, is known.

Figure 5 displays the value of $\text{corr}(u - P_2, P_2 - P_1 | \Xi_0)$ as a function of the correlation between per capita supply increments, ρ, and it is to be compared to Figure 4. One finds that, in this equilibrium, $B \neq 0$, a fact shown by the difference between (1) the value of ρ at the zero level of the short-dashed line in Figure 5 ($\rho = -0.16$) and (2) the analogous ρ in Figure 4 ($\rho = -0.42$). Furthermore, B varies with ρ and the other exogenous parameters. This suggests that empirical studies of the value of TA should not rely on the assumption that conditional expected price changes are uncorrelated.

3. Is the Equilibrium Weak-Form Efficient?

The fact that technical analysis has value in this market setting raises the question, "Is the 'market,' that is, the noisy rational expectations equilib-

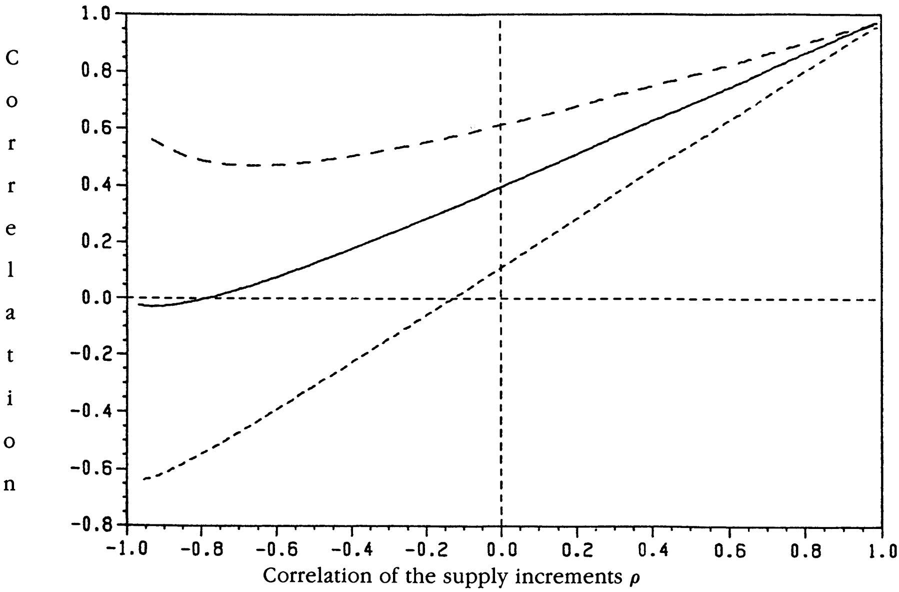

Figure 5
The unconditional correlation in price changes corr($P_2 - P_1$, $u - P_2|\Xi_0$), as a function of ρ, the correlation between supply increments x_1 and x_2. The solid line represents the case in which $V_1^2 = V_2^2 = 226$, the short-dashed line the case in which $V_1^2 = 113$ and $V_2^2 = 226$, and the long-dashed line the case in which $V_1^2 = 226$ and $V_2^2 = 113$. The values of the remaining parameters are set equal to the base values shown in Section A of Appendix C.

rium, weak-form efficient?" Not surprisingly, the answer depends on the definition of efficiency.

Fama (1970, 1976) defines a market as weak-form efficient if current prices "fully reflect" historical market statistics, including past prices. Here, "fully reflect" is taken to mean that conditioning on past and current prices provides the same set of beliefs as conditioning on only current price. The analysis of Section 2 demonstrates that, in this market, adding historical price to an investor's information set generally changes beliefs and investment policies. Thus, this market is inefficient in Fama's sense, even though the current price is determined by investors who rationally use past price in setting their demands.

Rubinstein's (1975) and Beaver's (1980) definitions require that publishing the information not change equilibrium prices. Rubinstein's notion allows one to ask only if the market is efficient with respect to all information, so it is of little interest to this discussion. Beaver provides a definition of market efficiency with respect to an information set that might be the information contained in historical prices. One finds that the noisy rational expectations equilibrium discussed here is weak-form efficient in Beaver's sense. But this is not surprising. *Any* perfectly competitive market in which the individuals have recall of past prices is weak-form efficient in Beaver's sense. Revelation of past prices does not change the current price or the distribution of future prices because historical prices are, *by assumption,* commonly known in equilibrium.[12]

Verrecchia (1982) defines a market to be efficient if, conditional on observation of the noise in the market, for example, x_1 and x_2, the price is no less efficient an estimator of the value of interest, for example, u, than the estimator available to any single trader. Each of the prices in the rational expectations equilibrium developed here reveals u perfectly when x_1 and x_2 are known. Hence, this market is efficient in this sense.

A market is efficient with respect to an information set in the sense of Latham (1986) if publishing that information does not change either prices or an individual's optimal policy. Considering the technical analyst of Section 2, one concludes that the noisy rational expectations equilibrium is not weak-form efficient in this sense. Adding historical price P_1 to this individual's information set, containing only current price P_2 and prior information Ξ_0, alters the optimal demand.

Discussions of weak-form efficiency in modern investment textbooks often include statements such as the following: "If the market is weak-form efficient, then current prices fully reflect the information in historical levels of prices and volumes of trade. Technical analysis has no value."[13] Such a statement is, in part, a warning to the reader to be skeptical of those offering investment advice based solely on naive analysis of historical data. Given the empirical work examining some (historically) popular forms of technical analysis, the advice apparently is well taken.[14]

What is not clear from textbook discussions of weak-form efficiency is whether the statement that "technical analysis has no value" is an implication, derived by logical reasoning, of the assumption that the market is weak-form efficient, or whether it is the defining characteristic of weak-form efficiency. If the latter is assumed, then the market discussed in this work is not weak-form efficient. Technical analysis does have value. Alternatively, in the case that "technical analysis has no value" is an implication of market efficiency, this work demonstrates that this inference may be unwarranted, given a definition under which the noisy rational expectations equilibrium is efficient.

4. Summary and Discussion of a Possible Extension

The noisy rational expectations model of Diamond and Verrecchia (1981) and Hellwig (1980) has been extended to two periods. It has been shown that TA has value in every myopic-investor economy. From the numerical analysis of the rational-investor economy, one finds that the second-period

[12] An alternative to the market studied here has historical prices excluded from investors' information sets. Such an alternative is not efficient in Beaver's sense; the addition of historical price information changes current prices.

[13] Examples of such statements include Jones (1985, p. 433), Reilly (1985, p. 197), and Haugen (1986, p. 469).

[14] See, for example, Alexander (1964), Fama and Blume (1966), Jensen (1967), and Seelenfreund et al. (1968). A caveat is that these are generally studies of unconditional moments; see Section 3.

price is dominated as an informative source by a weighted average of the first- and second-period prices. Investors use the historical price in determining time 2 demands because the current price does not reveal all publicly available information provided by price histories, that is, investors use technical analysis to their benefit.

Treynor and Ferguson (1985) study the "dual" of the problem presented here. The asserted usefulness of their methodology is based on the assumption that a trader knows the exogenously determined effect of a particular news item on an asset price given that the market has received the information. They examine the usefulness of past prices in estimating the time the news is disseminated. In their setting, a trader receiving the information privately must decide how to act. If he receives the information before the market, then he establishes the appropriate position to profit from the change in price that is forthcoming when the market becomes informed. If he receives the information after the market, then he does not act. The trader uses past prices to assess the probability that he has received information before the market.

The dynamic rational expectations equilibrium of plans, prices, and price conjectures studied here assumes that each trader knows the time of the release of private information (signals) to other traders but does not know the values of the signals. Past price levels, then, enable traders to make more precise inferences about the signals.

A logical, but not straightforward, extension of the present model allows the release time of the private information to be stochastic and heterogeneous across investors. For example, one might posit a subset of investors who are, with some positive probability, informed of, say, a firm's earnings prior to the public announcement of the earnings. In this case, the uninformed investors use historical price levels to forecast jointly the timing of the public announcement and the level of earnings.[15] In such a model, unlike that of Treynor and Ferguson, the effect of information on the price is endogenous. A caveat regarding the implementation of such a methodology is that the joint statistical behavior of price levels and earnings levels must be well understood. How well this is possible is, of course, an empirical question.

Appendix A: The Equilibrium Price and Demand Functions

It is shown in this appendix that the linear price conjectures (5) lead to 1) linear demand functions, Equations (6) or, alternatively, Equations (9) and 2) linear equilibrium price functions (8). It is also shown that conditions (10) are necessary for TA to have no value. Throughout, it is assumed that h_0, s_1, s_2, V_1, V_2, and R are each greater than zero; that $|\rho| < 1$; that the price coefficients γ_1 and δ_2 are nonzero; and that the values G_{11} and G_{12} (see Proposition A2) exist and are nonzero.

[15] Beaver, Lambert and Morse (1980) propose forming expectations regarding accounting earnings from stock price movements. However, they do not consider the possibility of serial correlation in price changes.

Proposition A1. *Given the conjectures (5), the coefficients of*

$$\mu_{i1} \equiv E(u \mid \Xi_{i1}) = ay_0 + by_{i1} + cP_1 \tag{A1}$$

$$E(x_1 \mid \Xi_{i1}) = ey_0 + fy_{i1} + gP_1 \tag{A2}$$

$$E(x_2 \mid \Xi_{i1}) = E(x_1 \mid \Xi_{i1})\frac{\rho V_2}{V_1} \tag{A3}$$

$$\eta_i \equiv E(P_2 \mid \Xi_{i1}) = \theta_1 y_0 + \theta_2 y_{i1} + \theta_3 P_1 \tag{A4}$$

$$\mu_{i2} \equiv E(u \mid \Xi_{i2}) = \lambda_1 \mu_{i1} + \lambda_2 y_{i2} + \lambda_3 (P_2 - \eta_i) \tag{A5}$$

and the quantities $VAR[u, y_{i2}, P_2) \mid \Xi_{i1}]$ *and* $\sigma_2^2 \equiv var(u \mid \Xi_{i2})$ *are constants conditional on* Ξ_0.

Proposition A1 follows from the properties of multivariate normal random variables. Computing an individual's time 1 demands in Proposition A3 below requires knowledge of the first row $G_1 \equiv [G_{11}, G_{12}]$ of the matrix $\mathbf{G} \equiv (2\mathbf{N} + \mathbf{W}^{-1})^{-1}$, where $\mathbf{W} \equiv \mathrm{VAR}[(P_2, \mu_{i2}) \mid \Xi_{i1}]$ and where $\mathbf{N}$ is defined by

$$\mathbf{N} \equiv \frac{1}{2}\begin{bmatrix} \sigma_2^{-2} & -\sigma_2^{-2} \\ -\sigma_2^{-2} & \sigma_2^{-2} \end{bmatrix}$$

Note that $\mathbf{W}$ is invertible whenever s_2, V_2^2, and δ_2 are nonzero.

Proposition A2. *Given conjectures (5),* G_{11} *and* G_{12} *are constants conditional on* Ξ_0.

This proposition follows from the definition of $\mathbf{N}$ and the properties of multivariate normal random variables.

Proposition A3. *The optimal strategy* (d_{i1}, d_{i2}) *for investor i given the conjectured price functionals is*

$$d_{i2} = \frac{\mu_{i2} - P_2}{R\sigma_2^2} \tag{A6}$$

$$d_{i1} = \frac{E(P_2 \mid \Xi_{i1}) - P_1}{RG_{11}} + \frac{E(d_{i2} \mid \Xi_{i1})(G_{12} - G_{11})}{G_{11}} \tag{A7}$$

Proof. The representation (A6) is the solution to the end-of-the-horizon problem [Equation (2a)] and its derivation can be found in Diamond and Verrecchia [1981, see their equation (10)]. Note that σ_2^2 is nonzero when V_1^2, V_2^2, s_1, s_2, γ_1, and δ_2 are nonzero.

Substituting Equation (A6) into the maximand of Equation (2a), one finds the time 1 derived utility of time 2 wealth is

$$J_{i2} = -\exp\left\{-R[n_0 + d_{i1}(P_2 - P_1)] - \frac{(1/2)(\mu_{i2} - P_2)^2}{\sigma_2^2}\right\}$$

Define $L_i' \equiv (-Rd_{i1}, 0)$, $M_i' \equiv (P_2, \mu_{i2})$, and $Q_i' \equiv E(M_i \mid \Xi_{i1})$. Then, using knowledge of multinormal variables,

$$\begin{aligned} E(J_{i2} \mid \Xi_{i1}) &= E[-\exp(-Rn_0 + d_{i1}P_1 + L_i'M_i - M_i'\mathbf{N}M_i) \mid \Xi_{i1}] \\ &= -|\mathbf{W}|^{-1/2}|2\mathbf{N} + \mathbf{W}^{-1}|^{-1/2}\exp[-Rn_0 + Rd_{i1}P_1 + L_i'Q_i - Q_i'\mathbf{N}Q_i \\ &\quad + (1/2)(L_i' - 2Q_i'\mathbf{N})(2\mathbf{N} + \mathbf{W}^{-1})^{-1}(L_i - 2\mathbf{N}Q_i)] \end{aligned}$$

The first-order condition with respect to d_{i1} is

$$P_1 - E(P_2 \mid \Xi_1) + d_{i1}RG_{11} - 2G_1'\mathbf{N}Q_i = 0$$

Algebra provides

$$d_{i1} = R^{-1}G_{11}^{-1}[E(P_{i2} \mid \Xi_1) - P_{i1}] + 2R^{-1}G_{11}^{-1}G_1'\mathbf{N}Q_i$$

Algebra and the definitions of G_1, $\mathbf{N}$, and Q_i provide Equation (A7). ■

Proposition A4. *Given conjectures (5), the optimal risky asset demands d_{i1} and d_{i2} can be written as linear functions:*

$$d_{i1} = \Psi_{11}y_0 + \Psi_{12}y_{i1} + \Psi_{13}P_1$$

and

$$d_{i2} = \Psi_{21}y_0 + \Psi_{22}y_{i1} + \Psi_{23}y_{i2} + \Psi_{24}P_1 + \Psi_{25}P_2$$

Proposition A4 follows from Propositions A1 and A3.

Proposition A5. *Given the price conjectures (5) and optimal demands (6), the price functions that satisfy the market-clearing conditions (3) are*

$$P_2 = \hat{\alpha}_2 y_0 + \hat{\beta}_2 u - \hat{\gamma}_2 x_1 - \hat{\delta}_2 x_2 \tag{A8}$$

and

$$P_1 = \hat{\alpha}_1 y_0 + \hat{\beta}_1 u - \hat{\gamma}_1 x_1 \tag{A9}$$

where the coefficients ($\hat{\alpha}_2$, $\hat{\beta}_2$, $\hat{\gamma}_2$, $\hat{\delta}_2$, $\hat{\alpha}_1$, $\hat{\beta}_1$, $\hat{\gamma}_1$) are constants conditional on Ξ_0 and are functions of the conjectured parameter values (without carats).

Proof. Given demands (6), which are derived in Proposition A3, market clearing at time 2 requires that P_2 satisfy

$$\begin{aligned} x_2 + x_1 &= d_2 \\ &= (\mu_2 - P_2)R^{-1}\sigma_2^{-2} \\ &= [\lambda_1\mu_1 + \lambda_2 u + \lambda_3(P_2 - \eta) - P_2]R^{-1}\sigma_2^{-2} \end{aligned} \tag{A10}$$

The second equality follows from Equation (A6) and the definitions of d_2 and μ_2. The third equality follows from Equation (A5) and the definitions of μ_1, y_2, and η. Relations (A1) and (A4) and the definitions of μ_1 and η provide

$$\mu_1 = ay_0 + bu + cP_1 \tag{A11}$$

and

$$\eta = \theta_1 y_0 + \theta_2 u + \theta_3 P_1 \tag{A12}$$

Rearranging Equation (A10) as a definition of P_2, then substituting for μ_1, η, and P_1 using Equations (A11), (A12) and conjectures (5b), respectively, provides the constant coefficients of Equation (A8).

Market clearing at time 1 requires

$$x_1 = d_1 = \frac{\eta}{RG_{11}} - \frac{P_1}{RG_{11}} + \frac{(\mu_1 - \eta)(G_{12} - G_{11})}{RG_{11}\sigma_2^2} \tag{A13}$$

The second equality follows from Equation (A7) and the definitions of μ_1 and η. Substituting in Equation (A13) for μ_1 and η using Equations (A11) and (A12), respectively, and algebra provides the constant coefficients of Equation (A9). ■

Proposition A6. *Technical analysis has value in equilibrium if and only if δ_2 and γ_2 are unequal. Furthermore, when δ_2 and γ_2 are equal in equilibrium, the ratios β_2/γ_2 and β_1/γ_1 must satisfy*

$$\frac{\beta_2}{\gamma_2} = \frac{s_2 + s_1}{Rs_1 s_2} \tag{A14}$$

$$\frac{\beta_1}{\gamma_1} = \frac{(s_1 + s_1)V_1(V_1 + \rho V_2)}{Rs_1 s_2(V_1^2 + \rho V_1 V_2 + V_2^2)} \tag{A15}$$

Proof. The expressions for $\hat{\gamma}_2$ and $\hat{\delta}_2$ derived from the market-clearing condition (A10) satisfy

$$\hat{\gamma}_2 - \hat{\delta}_2 = \gamma_1(H_2 c - Y_2\theta_3)(K_2 s_2^{-1} - Y_2)^{-1}$$

where $H_2 \equiv \text{var}(P_2|\Xi_{i1})$, $Y_2 \equiv \text{cov}(u, P_2|\Xi_{i1})$, $K_2 \equiv |\text{VAR}(y_{i2}, P_2|\Xi_{i1})|$, and c and θ_3 are defined in Equations (A1) and (A4), respectively. Hence in equilibrium (i.e., when $\hat{\gamma}_2 = \gamma_2$, etc.), γ_2 and δ_2 are unequal if and only if $H_2 c$ and $Y_2\theta_3$ are unequal. TA has value if and only if d_{i2} varies with P_1. Examination of the right side of Equation (A6) shows that d_{i2} varies with P_1 if and only if $K_2^{-1}s_2 H_2 c$ and $K_2^{-1}s_2 Y_2\theta_3$ are unequal; to see this result use Equations (A1), (A4), and (A5). Hence, TA has value if and only if γ_2 and δ_2 are unequal.

Given a ratio $Z \equiv \beta_1/\gamma_1$, the expressions for the time 2 coefficients satisfying Equation (A10) provide

$$\alpha_2 = 1 - \beta_2 \tag{A16}$$

$$\begin{aligned}\beta_2 D = {} & h_0 R^2 s_1^2 s_2^2 (V_1^2 + 2\rho V_1 V_2 + V_2^2) Z^2 \\ & - 2 h_0 R s_1 s_2 (s_1 + s_2)(V_1 V_2 \rho + V_1^2) Z \\ & + h_0 R^2 (1 - \rho^2) s_1 s_2 (s_1 + s_2) V_1^2 V_2^2 + h_0 (s_1 + s_2)^2 V_1^2\end{aligned} \tag{A17}$$

$$\begin{aligned}\gamma_2 D = {} & h_0 R^2 s_1^2 s_2^2 (V_1 V_2 \rho + V_2^2) Z \\ & + h_0 R^3 (1 - \rho^2) s_1^2 s_2^2 V_1^2 V_2^2 \\ & - h_0 R \rho s_1 s_2 (s_1 + s_2) V_1 V_2\end{aligned} \tag{A18}$$

$$\begin{aligned}\delta_2 D = {} & - h_0 R^2 s_1^2 s_2^2 (\rho V_1 V_2 + V_1^2) Z \\ & + h_0 R^3 (1 - \rho^2) s_1^2 s_2^2 V_1^2 V_2^2 + h_0 R s_1 s_2 (s_1 + s_2) V_1^2\end{aligned} \tag{A19}$$

where

$$\begin{aligned}D \equiv {} & h_0 R^2 s_1^2 s_2^2 (V_1^2 + 2\rho V_1 V_2 + V_2^2) Z^2 - 2 h_0 R s_1 s_2 (s_1 + s_2)(V_1 V_2 \rho + V_1^2) Z \\ & + R^2 (1 - \rho^2) s_1 s_2 V_1^2 V_2^2 [s_1 s_2 + h_0 (s_1 + s_2)] + h_0 (s_1 + s_2)^2 V_1^2\end{aligned}$$

Equating Equations (A18) and (A19) provides Equation (A15). Equality (A15) and the ratio of (A17) and (A18) provides (A14). ■

Appendix B. A Myopic-Investor Equilibrium

This appendix shows that an equilibrium exists and that technical analysis has value in every myopic-investor economy. The results of this appendix rely on the expressions for risky asset demands [Equations (6a) and (6c)]. These expressions are sensible only if $\mathrm{var}(P_2 | \Xi_{i1})$ and σ_2^2 are nonzero. The existence result, Theorem 1, demonstrates that γ_1 and δ_2 are each greater than zero in equilibrium and that this implies $\mathrm{var}(P_2 | \Xi_{i1}) > 0$ and $\sigma_2^2 > 0$. The results of this appendix also rely on the fact that each of the arguments of Appendix A is applicable to the myopic-investor economy except for the derivations of first-period demands and price coefficients.

Proof of Theorem 1

Given linear conjectures (5), the derivation of these coefficients of the price P_2 proceeds as it does in Proposition A5. These coefficients must satisfy Equations (A16) to (A19).

Given demands (6c), market clearing at time 1 requires

$$x_1 = d_1 = \frac{\eta - P_1}{R\,\mathrm{var}(P_2 \mid \Xi_{i1})} \tag{A20}$$

Using Equations (A20) and (A4) and the definition of η, one finds the coefficients of the time 1 price satisfy

$$\alpha_1 = 1 - \beta_1 \tag{A21}$$

$$\beta_1 = \frac{\theta_2}{1 - \theta_3} \tag{A22}$$

$$\gamma_1 = \frac{R\,\text{var}(P_2 \mid \Xi_{i1})}{1 - \theta_3} \tag{A23}$$

The values $\text{var}(P_2 | \Xi_{i1})$, θ_2, and θ_3 can be calculated as functions of the times 1 and 2 price coefficients; see Proposition A1. The ratio of Equations (A22) and (A23) implies that the ratio $Z \equiv \beta_1/\gamma_1$ satisfies

$$\begin{aligned} 0 = {} & h_0 R s_1(\delta_2^2 V_2^2 + 2\delta_2\gamma_2\rho V_1 V_2 + \gamma_2^2 V_2^2) Z^3 - 2 h_0 R s_1 \beta_2(\delta_2 \rho V_1 V_2 + \gamma_2 V_1^2) Z^2 \\ & + [\delta_2^2 R(1 - \rho^2)(s_1 + h_0) V_1^2 V_2^2 + \delta_2 h_0 \rho V_1 V_2^2 \\ & + \beta_2 h_0 R s_1^2 V_1 + \gamma_2 h_0 V_1^2] Z - \beta_2 h_0 V_1^2 \end{aligned} \tag{A24}$$

A linear rational expectations equilibrium exists if and only if Equation (A24) obtains for some real number Z and $\text{var}(P_2 | \Xi_{i1}) > 0$ and $\sigma_2^2 > 0$. From Equations (A17) to (A19), β_2, γ_2, and δ_2 are seen to be functions of Z. Upon substitution, Equation (A24) becomes a fifth-degree polynomial in Z which is hereby labeled $F(Z)$.

Proposition A1 implies that the variances $\text{var}(P_2 | \Xi_{i1})$ and σ_2^2 are positive when γ_1 and δ_2 are nonzero. Given the definition of Z, γ_1 is nonzero when Z is finite. Equilibrium exists if and only if a finite Z exists such that $F(Z) = 0$ and $\delta_2(Z) \neq 0$ where $\delta_2(Z)$ is given by Equation (A19).

We begin to argue the existence of a satisfactory Z by collecting a few results. The values $\beta_2(Z)$, $\gamma_2(Z)$, $\delta_2(Z)$, that is, Equations (A17) to (A19), are each finite when $Z = 0$. Also $\beta_2(0) > 0$. Hence $F(0) = -\beta_2 h_0 V_1^2 < 0$. Let Z^0 satisfy $\delta_2(Z^0) = 0$; note that if $\rho V_2 + V_1 = 0$, then Z^0 does not exist, otherwise $\text{sign}(Z^0) = \text{sign}(V_1 + \rho V_2)$. Also, when Z^0 exists, $\beta_2(Z^0) = Z^0 \gamma_2(Z^0)$ and, therefore, $F(Z^0) = 0$. Differentiating $F(Z)$, one finds

$$F'(Z^0) = \frac{\begin{aligned} & h_0 R^3 (1 - \rho)^2 (1 + \rho)^2 s_1^2 s_2^2 V_1^5 V_2^4 \\ & \cdot \{ h_0 [R^2 s_1 s_2 (V_1^2 + 2\rho V_1 V_2 + V_2^2) + s_1 + s_2] \\ & [R^2 s_1 s_2 V_2^2 (\rho - 1) - s_1 - s_2] - R^2 s_1^2 s_2^2 (V_1 + \rho V_2)^2 \} \end{aligned}}{V_1 + \rho V_2}$$

The numerator is negative so that $\text{sign}[F'(Z^0)] = -\text{sign}(V_1 + \rho V_2)$. Finally, $\lim_{Z\to\infty} \beta_2(Z) = 1$ and $\lim_{Z\to\infty} \gamma_2(Z) = \lim_{Z\to\infty} \delta_2(Z) = 0$, so that $\lim_{Z\to\infty} F(Z) = \infty$.

Because $F(Z)$ is continuous in Z, $F(0) < 0$ and $\lim_{Z\to\infty} F(Z) = \infty$, the intermediate value theorem [Rosenlicht (1968, p. 82)] implies the existence of $0 < Z^* < \infty$ such that $F(Z^*) = 0$. If $V_1 + \rho V_2 < 0$, then $Z^0 < 0$ and, using the linearity of $\delta_2(Z)$, $\delta_2(Z^*) \neq 0$, so an equilibrium exists. If $V_1 + \rho V_2 = 0$, then $\delta_2(Z) \neq 0$, for all Z, so an equilibrium exists. If $V_1 + \rho V_2 >$

0, then $F(Z^0) = 0$, $F'(Z^0) < 0$, and $Z^0 > 0$. Hence, using the intermediate value theorem and the linearity of $\delta_2(Z)$, there exists $0 < Z^* < Z^0$ such that $F(Z^*) = 0$ and $\delta_2(Z^*) \neq 0$. It follows that a linear, two-period rational expectations equilibrium exists when investors are myopic. ■

Proof of Theorem 2

Expression (A15) for the ratio $Z \equiv \beta_1/\gamma_1$ must obtain whenever TA has no value in the myopic-investor equilibrium. The ratio Z must simultaneously solve Equation (A24) in an equilibrium. Equality (A24), upon substitution of Equation (A15) into its right-hand side, becomes

$$0 = \frac{h_0(1-\rho^2)(s_1^2+s_2)V_1V_2[s_1^2+s_2+R^2s_1s_2(V_2^2+2\rho V_1V_2^2+V_1^2)]T}{s_2(V_2^2+2\rho V_1V_2+V_1^2)^2\{R^2s_1s_2[s_1s_2+h_0(s_1+s_2)] \cdot (V_1^2+2\rho V_1V_2+V_2^2)+h_0(s_1+s_2)^2\}^2} \tag{A25}$$

where
$$\begin{aligned}
T &\equiv h_0(R^2T_1+T_2)(R^2T_3+T_4)+R^2T_5(R^2T_6+T_7)\\
T_1 &\equiv s_1s_2(V_1^2+2\rho V_1V_2+V_2^2)\\
T_2 &\equiv s_1+s_2\\
T_3 &\equiv s_1s_2^2(V_1^2+\rho V_1V_2)(V_1^2+2\rho V_1V_2+V_2^2)\\
T_4 &\equiv (s_1+s_2)[s_1V_1^2+\rho V_1V_2(s_1-s_2)-s_2V_2^2]\\
T_5 &\equiv s_1^2s_2^2\,(V_1^2+2\rho V_1V_2+V_2^2)\\
T_6 &\equiv s_1s_2\,(V_1^2+\rho V_1V_2)(V_1^2+2\rho V_1V_2+V_2^2)\\
T_7 &\equiv s_1V_1^2+\rho V_1V_2(s_1-s_2)-s_2V_2^2
\end{aligned}$$

Label as Θ that subset of $\Re^7$ meeting the parametric restrictions, that is, s_1, s_2, V_1^2, V_2^2, h_0, and R each greater than zero and $|\rho| < 1$. Note that Equation (A25) obtains if and only if $T = 0$. Also note that each of the values $(R^2T_1 + T_2)$, $(R^2T_3 + T_4)$, R^2T_5, and $(R^2T_6 + T_7)$ is never zero on Θ. Thus, T is zero on Θ if and only if h_0 is equal to

$$\underline{h} \equiv \frac{-R^2T_5(R^2T_6+T_7)}{(R^2T_1+T_2)(R^2T_3+T_4)} \tag{A26}$$

The value of $\underline{h}$ is never zero on Θ and, because each of the T_j is continuous on Θ, $\underline{h}$ is continuous on Θ. Inspection of $\underline{h}$ shows its value is negative whenever $V_1 = V_2$ and $s_1 = s_2$. Therefore, h_0 and $\underline{h}$ cannot be equal and, consequently, TA has value at all points in Θ. ■

Appendix C. Numerical Example of Equilibrium in a Two-Period Rational Expectations Economy

A. Base values of parameters

Risk aversion	$R = 1.0$
Correlation between supply increments	$\rho = 0$
Prior variance of risky asset payoff	$h_0 = 0.4623$
Prior expected value of risky asset payoff	$y_0 = 2.79$

	Time 1	Time 2
Variance of per capita supply increment	$V_1^2 = 226.0$	$V_2^2 = 226.0$
Variance of the private signal error	$s_1 = 0.2529$	$s_2 = 0.2529$

B. Equilibrium price functions

$$P_2 = 0.2106y_0 + 0.7894u - 0.0979x_1 - 0.1002x_2$$
$$P_1 = 0.5325y_0 + 0.4675u - 0.3557x_1$$

C. Equilibrium risky asset demand functions

$$d_{i2} = 2.1358y_0 + 3.9154y_{i1} + 3.9154y_{i2} - 0.0638P_1 - 9.9802P_2$$
$$d_{i1} = 1.4970y_0 + 1.3142y_{i1} - 2.8112P_1$$

D. The conditional expected payoff

$$\mu_{i1} \equiv E(u|\Xi_{i1}) = 0.3518y_0 + 0.6456y_{i1} + 0.0026P_1$$
$$\mu_{i2} \equiv E(u|\Xi_{i2}) = 0.2079y_0 + 0.3850y_{i1} + 0.3850y_{i2} - 0.0062P_1 + 0.0284P_2$$

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